

# Sommerfeld identity

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The **Sommerfeld identity** is a mathematical identity, due Arnold Sommerfeld, used in the theory of propagation of waves,

$$\frac{e^{ikr}}{r} = \int_0^{\infty} I_0(\lambda r) \frac{\lambda d\lambda}{\mu}$$

where

$$\mu = \sqrt{\lambda^2 - k^2}$$

is to be taken with positive real part, to ensure the convergence of the integral. The function  $I_0$  is a Bessel function. Here the notation for Bessel functions follows the German convention, to be consistent with the original notation used by Sommerfeld. In English literature it is more common to use

$$I_n(\rho) = J_n(i\rho).$$

This identity is known as the **Sommerfeld Identity** [Ref.1].

An alternative form is

$$\frac{e^{ik_0 r}}{r} = i \int_0^{\infty} dk_{\rho} \frac{k_{\rho}}{k_z} J_0(k_{\rho} \rho) e^{ik_z |z|}$$

Where

$$k_z = (k_0 - k_{\rho})^{1/2}$$

[Ref.2,Pg.66].

The physical interpretation is that a spherical wave can be expanded into a summation of cylindrical waves in  $\rho$  direction, multiplied by a plane wave in the  $z$  direction. The summation has to be taken over all the wavenumbers  $k_{\rho}$ .

## References

1. Sommerfeld, A., *Partial Differential Equations in Physics*, Academic Press, 1964
2. Chew, W.C., *Waves and Fields in Inhomogenous Media*, Van Nostrand Reinhold, New York, 1990

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